DOCUMENT RESUME

ED 038 294 24 SE 008 178

AUTHOR	Meyer, Rochelle Wilson
TITLE	The Identification and Encouragement of Mathematical Creativity in First Grade Students, (Part 3).
INSTITUTION	Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
REPORT NO	TR-112
BUREAU NO	BR-5-0216
PUB DATE	Jan 70
CONTRACT	OEC-5-10-154
NOTE	84p.
EDRS PRICE	EDRS Price MF-$0.50 HC-$4.30
DESCRIPTORS	*Creative Thinking, Creativity, Doctoral Theses, *Elementary School Mathematics, *Grade 1, *Learning, Mathematical Concepts, Mathematical Experience, *Research

ABSTRACT

Reported is research involving the development and testing of a program designed to encourage individual creative mathematical activity in first grade students. Initially, some characteristics of the creative process and creative thinking were examined and six criteria describing certain aspects of mathematical creativity were identified and validated. An instrument used to measure observable mathematical creativity was designed in order to test the program on mathematical creativity. Two hypotheses were formulated: H1: "Participation in the program will increase a student's observable mathematical creativity"; H2: "Participation in the program will not affect a student's performance on a test of general creative ability". Part III contains the appendices of this report. (FL)

ct No. 112 (Part III)

THE IDENTIFICATION AND ENCOURAGEMENT OF MATHEMATICAI CREATIVITY IN FIRST GRADE STUDENTS

Report from the Project on
Analysis of Mathematics Instruction

U.S. Office of Education Center No. C-03 Contract OE 5-10-154

Technical Report No. 112 (Part III)

THE IDENTIFICATION AND ENCOURAGEMENT OF MATHEMATICAL CREATIVITY IN FIRST GRADE STUDENTS

Report from the Project on
Analysis of Mathematics Instruction

By Rochelle Wilson Meyer

John G. Harvey, Professor of Curriculum & Instruction & Mathematics
Chairman of the Examining Committee

John G. Harvey and Thomas A. Romberg, Principal Investigators

Wisconsin Research and Development
Center for Cognitive Learning
The University of Wisconsin
Madison, Wisconsin

January 1970

Published by the Wisconsin Research and Development Center for Cognitive Learning, supported in part as a research and development center by funds from the United States Office of Education, Department of Health, Education, and Welfare. The opinions expressed herein do not necessarily reflect the position or policy of the Office of Education and no official endorsement by the Office of Education should be inferred.

Center No. C-03 / Contract OE 5-10-154

NATIONAL EVALUATION COMMITTEE

Samuel Brownell
Professor of Urban Education
Graduate School
Yale University

Henry Chauncey
President
Educational Testing Service

Elizabeth Koontz
President
National Education Association

Patrick Suppes
Professor
Department of Mathematics
Stanford University

Launor F. Carter
Senior Vice President on Technology and Development
System Development Corporation

Martin Deutsch
Director, Institute for Developmental Studies
New York Medical College

Roderick McPhee
President
Punahou School, Honolulu

***Benton J. Underwood**
Professor
Department of Psychology
Northwestern University

Francis S. Chase
Professor
Department of Education
University of Chicago

Jack Edling
Director, Teaching Research Division
Oregon State System of Higher Education

G. Wesley Sowards
Director, Elementary Education
Florida State University

UNIVERSITY POLICY REVIEW BOARD

Leonard Berkowitz
Chairman
Department of Psychology

John Guy Fowlkes
Director
Wisconsin Improvement Program

Herbert J. Klausmeier
Director, R & D Center
Professor of Educational Psychology

M. Crawford Young
Associate Dean
The Graduate School

Archie A. Buchmiller
Deputy State Superintendent
Department of Public Instruction

Robert E. Grinder
Chairman
Department of Educational Psychology

Donald J. McCarty
Dean
School of Education

***James W. Cleary**
Vice Chancellor for Academic Affairs

H. Clifton Hutchins
Chairman
Department of Curriculum and Instruction

Ira Sharkansky
Associate Professor of Political Science

Leon D. Epstein
Dean
College of Letters and Science

Clauston Jenkins
Assistant Director
Coordinating Committee for Higher Education

Henry C. Weinlick
Executive Secretary
Wisconsin Education Association

EXECUTIVE COMMITTEE

Edgar F. Borgatta
Brittingham Professor of Sociology

Russell J. Hosler
Professor of Curriculum and Instruction and of Business

Wayne Otto
Professor of Curriculum and Instruction (Reading)

Richard L. Venezky
Assistant Professor of English and of Computer Sciences

Max R. Goodson
Professor of Educational Policy Studies

***Herbert J. Klausmeier**
Director, R & D Center
Professor of Educational Psychology

Robert G. Petzold
Associate Dean of the School of Education
Professor of Curriculum and Instruction and of Music

FACULTY OF PRINCIPAL INVESTIGATORS

Ronald R. Allen
Associate Professor of Speech and of Curriculum and Instruction

Gary A. Davis
Associate Professor of Educational Psychology

Max R. Goodson
Professor of Educational Policy Studies

Richard G. Morrow
Assistant Professor of Educational Administration

Vernon L. Allen
Associate Professor of Psychology
(On leave 1968-69)

M. Vere DeVault
Professor of Curriculum and Instruction (Mathematics)

Warren O. Hagstrom
Professor of Sociology

Wayne Otto
Professor of Curriculum and Instruction (Reading)

Nathan S. Blount
Associate Professor of English and of Curriculum and Instruction

Frank H. Farley
Assistant Professor of Educational Psychology

John G. Harvey
Associate Professor of Mathematics and Curriculum and Instruction

Milton O. Pella
Professor of Curriculum and Instruction (Science)

Robert C. Calfee
Associate Professor of Psychology

John Guy Fowlkes (Advisor)
Professor of Educational Administration
Director of the Wisconsin Improvement Program

Herbert J. Klausmeier
Director, R & D Center
Professor of Educational Psychology

Thomas A. Romberg
Assistant Professor of Mathematics and of Curriculum and Instruction

Robert E. Davidson
Assistant Professor of Educational Psychology

Lester S. Golub
Lecturer in Curriculum and Instruction and in English

Burton W. Kreitlow
Professor of Educational Policy Studies and of Agricultural and Extension Education

Richard L. Venezky
Assistant Professor of English and of Computer Sciences

MANAGEMENT COUNCIL

***Herbert J. Klausmeier**
Director, R & D Center
Acting Director, Program 1

Thomas A. Romberg
Director
Programs 2 and 3

James E. Walter
Director
Dissemination Section

Dan G. Woolpert
Director
Operations and Business

Mary R. Quilling
Director
Technical Section

* COMMITTEE CHAIRMAN

STATEMENT OF FOCUS

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from Phase 2 of the Project on Prototypic Instructional Systems in Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

To my parents

TABLE OF CONTENTS

Chapter		Page
	List of Figures	vii
	List of Tables	viii
	Acknowledgements	x
	Abstract	xii
I.	BACKGROUND	1
	1.1 A BRIEF STATEMENT OF THE PROBLEM	1
	1.2 OUTLINE OF THE THESIS	1
	1.3 OUTLINE OF CHAPTER I	3
	1.4 THE IMPORTANCE OF MATHEMATICAL CREATIVITY	3
	1.5 THE CREATIVE PROCESS: A SEQUENCE OF STAGES	5
	1.6 THE CREATIVE PERSON	14
	1.7 MATHEMATICAL CREATIVITY: A SPECIAL CASE	21
II.	CLASSROOM CONDITIONS WHICH ENCOURAGE MATHEMATICAL CREATIVITY	30
	2.1 OUTLINE OF CHAPTER II	30
	2.2 THE MATHEMATICAL SITUATION	30
	2.3 THE ACTIONS OF THE TEACHER	33
	2.4 THE DURATION AND SPACING OF THE LESSONS	41
III.	PILOT STUDIES	43
	3.1 OUTLINE OF CHAPTER III	43
	3.2 DESCRIPTION OF LAKE MILLS AND THE PROSPECT STREET SCHOOL	44
	3.3 THE FIRST PILOT STUDY	44
	3.4 THE SECOND PILOT STUDY	50
IV.	THE EXPERIMENTAL PROGRAM	63
	4.1 OUTLINE OF CHAPTER IV	63
	4.2 THE ACTIVITIES CHOSEN	63
	4.3 DESCRIPTION OF POYNETTE AND THE ELEMENTARY SCHOOL	68
	4.4 THE ACCOUNT OF THE EXPERIMENTAL PROGRAM	69

TABLE OF CONTENTS (Continued)

Chapter		Page
V.	THE TEST INSTRUMENT	81
	5.1 OUTLINE OF CHAPTER V	81
	5.2 THE CRITERIA	81
	5.3 THE MATHEMATICS PROBLEMS USED AS PRETEST AND POSTTEST	87
	5.4 SCORING THE VIDEOTAPES	92
VI.	THE EXPERIMENT	109
	6.1 OUTLINE OF CHAPTER VI	109
	6.2 RESTATEMENT OF THE PROBLEM	109
	6.3 THE EXPERIMENTAL DESIGN	112
	6.4 THE DATA	118
VII.	CONCLUSIONS	142
	7.1 OUTLINE OF CHAPTER VII	142
	7.2 THE TEST INSTRUMENT	142
	7.3 THE EXPERIMENT	150
	7.4 SUMMARY	158
REFERENCES		162
Appendix A:	JOURNAL OF THE EXPERIMENTAL PROGRAM	165
Appendix B:	FACE VALIDATION MATERIALS AND RESULTS	205
Appendix C:	MATHEMATICAL PROBLEMS USED FOR PRETEST AND POSTTEST	221
Appendix D:	SCORING THE VIDEOTAPES	227

LIST OF FIGURES

FIGURE		PAGE
3.1	Two Responses to the Problem of Six Straws	48
3.2	Patterned Papers Used in the Second Pilot Study	56
4.1	Summary Chart Made During Activity One	73
4.2	Summary Chart Made During Activity Five	79
5.1	Three Trapezoids Used in the Posttest Problem	91
A.1	Proposed Classification Chart for Activity One	169
A.2	Classification Chart Made During Activity One	173
A.3	Two Coverings Having Horizontal Rows	178
A.4	A Covering Having Vertical Rows	179
A.5	A Covering Using All Three Shapes	181
A.6	An Attempted Covering By Squares	182
A.7	Summary Chart Made During Activity One	183
A.8	Proposed Summary Chart for Activity Five	201
A.9	Summary Chart Made During Activity Five	202
C.1	Arrangement of the Table at the Pretest	222
C.2	Three Trapezoids Used in the Posttest Problem	225
C.3	Arrangement of the Table at the Posttest	225

LIST OF TABLES

TABLE		PAGE
5.1	CRITERION CONSIDERED SATISFIED BY A MAJORITY SCORERS.	100
5.2	PRETEST: ERROR RATES OF THE SCORERS	102
5.3	POSTTEST: ERROR RATES OF THE SCORERS.	104
5.4	OVERALL: ERROR RATES OF THE SCORERS	105
5.5	INTERSCORER AGREEMENT.	107
5.6	INTERSCORER AGREEMENT OF SCORERS 1, 2, 4, AND 5.	108
6.1	SOLOMON FOUR-GROUP DESIGN.	113
6.2	INDIVIDUAL SCORES AND GROUP AVERAGES	115
6.3	ANALYSIS OF VARIANCE: H1.1a	122
6.4	ANALYSIS OF VARIANCE: H1.1b	123
6.5	ANALYSIS OF VARIANCE: H1.1c	124
6.6	ANALYSIS OF VARIANCE: H1.1d	125
6.7	ANALYSIS OF VARIANCE: H1.2a	126
6.8	ANALYSIS OF VARIANCE: H1.2b	127
6.9	ANALYSIS OF VARIANCE: H1.3a	128
6.10	ANALYSIS OF VARIANCE: H1.3b	129
6.11	ANALYSIS OF VARIANCE: H2.1a	132
6.12	ANALYSIS OF VARIANCE: H2.1b	133
6.13	ANALYSIS OF VARIANCE: H2.1c	134
6.14	ANALYSIS OF VARIANCE: H2.2a	135
6.15	ANALYSIS OF VARIANCE: H2.2b	136
6.16	ANALYSIS OF VARIANCE: H2.2c	137

LIST OF TABLES (CONTINUED)

TABLE		PAGE
6.17	ANALYSIS OF VARIANCE: H2.3a.	138
6.18	ANALYSIS OF VARIANCE: H2.3b.	139
6.19	ANALYSIS OF VARIANCE: H2.3c.	140
6.20	CORRELATION MATRIX OF POSTTEST SCORES	141
B.1	RESULTS OF FACE VALIDATION.	219

ACKNOWLEDGEMENTS

Many people have helped in the research reported in this thesis. Although it is impossible to mention everyone here, some people deserve special recognition for their assistance.

Dr. John G. Harvey has been what good major professors are supposed to be--he has been of valuable help to me both in formulating the research and in writing the thesis. I was not once but twice fortunate; in addition to Dr. Harvey I was able to benefit from the advice of Dr. Thomas A. Romberg, who has given of his time and ideas as if I were his own student. These two, plus Dr. Joshua Chover and Dr. Gary A. Davis, read the drafts of the thesis and made many good suggestions.

Two pilot studies were graciously accepted into the Prospect Street Elementary School, Lake Mills, Wisconsin. For their cooperation I thank Mr. Ronald Hering, Director of Research Activities; Mr. Thomas Block, Principal; Mrs. Linda Bender, Mrs. Marianna Buchanan, Miss Cheryl Hagan, and Mrs. Inez Shultz, teachers. A second gracious acceptance allowed the experiment to be conducted in the Poynette Elementary School, Poynette, Wisconsin. I am indebted to Mr. Gerald Makie, Superintendent; Mr. Glenn Porterfiled, Principal; Mr. Charles Tucker, Elementary Supervisor; and Mrs. Hansen, the teacher, for their warm welcome and cooperation. Mr. George Glasrud located these two friendly school districts and made the initial contacts.

For taking time out of their busy schedules to participate in the face validation I thank Dr. Richard A. Brualdi, Dr. Joshua Chover, Dr. Donald Crowe, Dr. Simon Hellerstein, Dr. Mary Ellen Rudin, Dr. Hans Schneider, and Dr. Melvin C. Thornton.

Mr. John McFee did the videotaping and editing and managed to remain cheerful and interesting through it all.

Mrs. Carolyn Gornowicz did such a fine job of teaching during the experiment that I could relax and enjoy the students, knowing that they were in good hands.

Mr. Thomas Fischbach helped in some difficult problems of statistical analysis.

Mrs. Cindy Gaa not only typed the manuscript but pointed out missing verbs and commas along the way.

This research was supported by the United States Office of Education, Department of Health, Education, and Welfare, under the provisions of the Cooperative Research Program, through a contract with the Wisconsin Research and Development Center for Cognitive Learning (Center No. C-03/Contract OE 5-10-154).

My husband, Walter Meyer, has helped in ways that words cannot adequately express--ways that are in some sense so subtle that I find new aspects creeping into my awareness every day. All this while he too was writing a thesis.

And last, the students who took part in the pilot studies and the experiment and whose honest and creative reactions to the mathematics problems were wonderful and rewarding to watch.

ABSTRACT

The importance of mathematical creativity is widely acknowledged. The initial research was an examination of some characteristics of the creative process and the creative person. On the basis of this background, six criteria describing observable aspects of mathematical creativity were identified. These criteria were face validated by seven Professors of Mathematics at the University of Wisconsin and serve as part of a test instrument to measure observable mathematical creativity. One set of conditions conducive to mathematical creativity was proposed and activities which satisfy these conditions were piloted. From these activities both an instructional program to encourage individual mathematical creativity in first grade students and two problems to use a part of the test instrument were developed. An experiment was conducted to determine the effects of participation in the program on observable mathematical creativity; these effects were measured using the test instrument developed for this thesis. The effects on general creativity were measured using the Torrance Tests of Creative Thinking, Figural Forms A and B. The major contributions of this thesis are the identification and face validation of six criteria which describe observable aspects of mathematical creativity and the presentation of evidence that under suitable conditions first grade students can exhibit behaviors satisfying these criteria.

APPENDIX A

JOURNAL OF THE EXPERIMENTAL PROGRAM

JOURNAL OF THE EXPERIMENTAL PROGRAM

In this journal, the events of the experimental program are reported. For each day the plan for the lesson, the events of the lesson and an evaluation are given. The days are grouped by the main activities pursued; for each activity, the materials used and an outline of the planned lessons are presented.

During each class period, all six students sat around one large table. A laboratory timer was used to signal the end of the twenty minutes.

ACTIVITY 1: TILING

Materials: Collection of approximately 100 each of painted cardboard tiles in one of three shapes: red equilateral triangles, blue squares, green diamonds (rhombi formed from two equilateral triangles); each tile $1\frac{1}{2}$ inches per side.

Unlined white paper, $5\frac{1}{2} \times 8\frac{1}{2}$ inches.

Ball point pens.

For a chart, manilla paper 11 x 18 inches; black felt-tip pen; triangles, squares and diamonds cut from construction paper of a matching color; tape.

The activity was to start with finding ways to completely cover a sheet of paper with the tiles so that the tiles do not overlap. Then the set of all possible coverings was to be classified according to the shapes used in each and this information recorded on a chart. Using the chart as a guide, the class was to try to make examples of each specific type of covering; in order to determine whether each type of covering had been made, a means of recording what the coverings looked like without using up the tiles was to be devised.

Day 1: Plan

The teacher greets the students, introduces herself and her aide, the author, and ask each student for his name. She then gives each student a sheet of paper and a pen and asks that each print his name on the paper and leave it in front of him so that she can look at the paper to help her remember the names. Then the teacher says, "I will be meeting you every day at this time for about three weeks. I have brought many interesting things for you to do; I hope you like them."

Next the teacher puts the three opened coffee cans which contain the shapes on the table, reaches into one and picks up a tile which she holds where the students can see it and asks, "Does anyone know the name of this shape?" It is expected that at least one of the students will suggest an acceptable name for each shape. The acceptable names for each shape are, in order of preference, triangle; square, box, rectangle; diamond, kite, tilted square. The teacher should respond to a preferred name by saying, "Yes, this is a (name of shape)," and then by tipping the can to show that all the other tiles in the can are the same as the sample. "This can has many (color) (name of shape) in it."

The teacher now explains and demonstrates the task. "I am giving each of you another piece of paper. Do you think you could cover the paper with these tiles? You can use as many of each kind of shape as you want, but the tiles must not overlap, like this, and there must be no paper showing through when you are done. If there is paper big enough to touch with your pinky, then that's too much paper showing. You might have to go over the edge of the paper with a tile - that's allowed." The teacher repeats the directions until all understand, then distributes the tiles in piles so all students can reach all shapes, and lets the students begin.

If a student finishes one covering, the teacher should invite him to make another "different" one. When each student has made at least one covering, the teacher should call for a pause in the work so that "everyone can see what we each have done." The teacher should point to two coverings and ask, " What ways are these two coverings the same; what ways are they different?" The students will probably not understand at first how to respond to this kind of comparison question, and may need some priming, such as "This covering has squares in it and so does this one, so one way the coverings are the same is that they both have squares. Lets see what other coverings have squares too." When all the coverings have been examined on one property, such as "has squares," the teacher should repeat her original comparison question in reference to the first two coverings. This time the students may suggest ways to compare the coverings. If not the teacher should suggest properties such as "has triangles," and "has diamonds."

When no more suggestions for comparison come from the students and the three properties - has squares, has triangles, and has diamonds - have been discussed, the teacher should say, "Let's put some of our ideas down in a chart." Then she gets out the materials for the chart and says, while constructing the chart, "We could call our chart 'Shapes we could put in a covering.' Now let's see, what shapes are in your covering (student's name)? Okay, one kind of covering has triangles and squares. What's another kind?" The teacher should try to get the students to suggest the seven kinds and should put the shapes designating each kind on the chart in the order in which the kinds are suggested. If a possibility is missing, say triangle only, but an analogous one, say square only, is already on the chart, the teacher should say, "We have square only, do we have that kind of covering for all the shapes? Which ones are missing?" At the end of this, the chart should look something like Figure A.1.

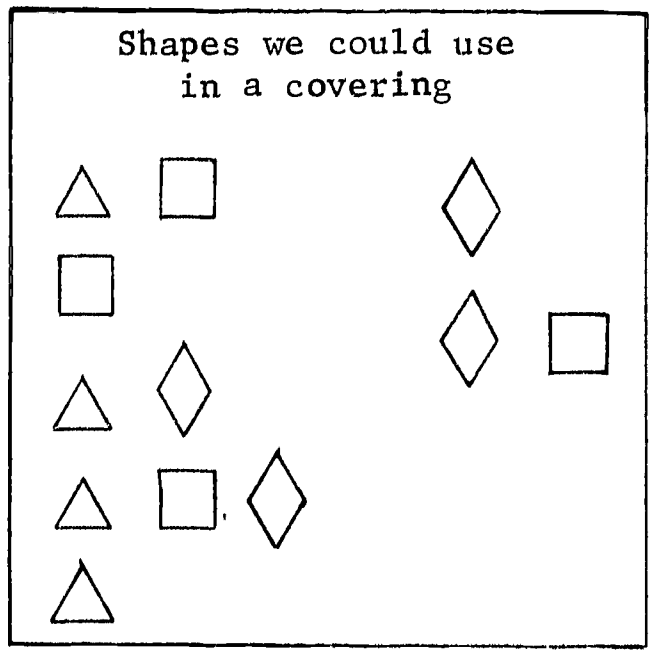


Figure A.1

Proposed Classification Chart for Activity One

Day 1: Events

The introduction and explanation of the task went as expected. The students knew the names "triangle," "square," and "diamond." The only change in plan was an addition made by Mrs. Gornowicz to explain the function of the laboratory timer.

When the covering activity began, five of the six students started to cover their papers with squares. A fortunate accident in the materials presented itself as a problem rather quickly - there were not enough squares for all five students. The teacher listened as the students asked for more squares, then explained that they already had all there were, and that they would have to use the other shapes to finish covering their papers. Three different types of coverings were attempted: coverings with all three shapes, coverings with triangles and squares, and coverings with triangles and diamonds. The variety with no squares was the type started by the sixth student while the others were running out of squares; the two students who made two coverings both made this variety as the second covering. All the students who had started with just squares added to the squares to finish that covering.

During the process of adding triangles or diamonds to partial coverings of squares, several students tried to fill in a ninety degree angle using a sixty degree angle or a one hundred twenty degree angle. Most students needed to be encouraged to remove some tiles already on the covering in order to make the new shapes fit better. One student never did complete a covering.

The timer sounded as the comparison activity was beginning. That activity was somewhat rushed and took the form of the teacher pointing

to two coverings, asking either for similarities or for differences and then going on to another pair of coverings. The responses were primarily spontaneous and in terms of colors present in the coverings. Whenever a color was mentioned, the teacher would ask a quesion like, "What's another name for the blue ones?" Then the students would respond "square." One student remarked that two coverings were similar in that they both had "blue on the top and red on the bottom."

Day 1: Evaluation

The lesson went well, but not as much was covered as had been planned. It was decided that Day 2 should begin with a review of the activity of Day 1 and then the chart should be made.

The inadequate supply of squares was considered a lucky accident which could not have been planned better than it was, since the problem was imposed by a limitation of the materials and not by authority of the teacher.

One student seemed highly nervous and frustratable during the lesson. It was decided to try to refrain from making any negative evaluative comments to her in order to try to bolster her confidence.

Day 2: Plan

The teacher starts by asking the students what they did the previous day. She lets them describe the activity, and asks questions as necessary so that the number and names of the shapes used and the rules by which the coverings were made are all restated. Then each student should be

asked which shapes he used in his covering. Next the teacher says, "Let's make a chart . . . ." (see Day 1: Plan).

When the chart is done, the teacher asks the aide to hang it on the wall. Then she asks, "Have we seen coverings of each kind that is on the chart? What kinds have we made already? Let's see if we can get at least one of every kind. Each of you pick the kind you're going to make. Then on this piece of paper I'm giving you, draw a picture of each shape that will be in your coverings. This piece of paper will be like a promise - every shape on it must be in your covering and you can't use a shape that is not on your paper. If you change your mind, draw a new promise paper. Here is paper for covering."

Then the teacher passes out the tiles and the covering begins. She should try to encourage each student to try a different type. One way is to ask the student how many types of coverings there are (seven) and how many students (six) so could each student do a different kind?

Day 2: Events

The review went as planned. The students were so excited at the suggestion of making a chart that they clapped. Mrs. Gornowicz had the students review again the kinds of coverings they had made as she constructed the chart. When the kinds which were possible but had not yet been made were discussed, the students needed to be reminded of the kind "lots of people tried yesterday but we ran out of something," - square only. Mrs. Gornowicz needed to make two columns of the groups of shapes. She asked the students if that would confuse them and they suggested that she number the kinds and draw a line between the columns so no one

would get confused. After each kind was put on the chart, Mrs. Gornowicz asked, "Any more?" She also asked this after the seventh and final kind. Then she asked "What can we call our chart?" The students suggested names like "Shapes," to which the teacher responded "What about the shapes?" or "What did we do with the shapes?" until a reasonable title was decided upon. The finished chart looked like Figure A.2.

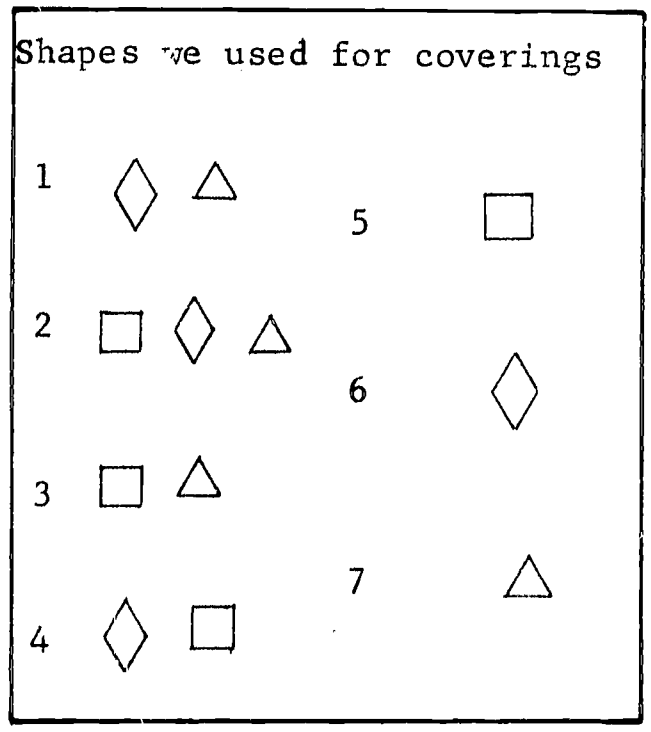


Figure A.2

Classification Chart Made During Activity One

The explanation of the promise papers was understood. After having drawn their promise papers, several students announced plans for their coverings such as, "I'm going to make all triangles on the top and all squares on the bottom." Not much time was left for making the coverings; most students barely finished one.

Day 2: Evaluation

Mrs. Gornowicz made three good modifications of the lesson plans: she asked for more kinds of coverings when the seven kinds had been exhausted, she let the students suggest a means of avoiding a confusing arrangement on the chart and she had the students help choose the title of the chart. These modifications were all in the spirit of encouraging creativity and indicated to the author that sometimes Mrs. Gornowicz understood the techniques the author wanted used in the classroom more completely than they were spelled out in the lesson plans. Because of this, the lesson plans became more abbreviated, the author relying on Mrs. Gornowicz to know how to review an activity or present a problem.

It had become evident that the three lessons allotted to Activity One would not be sufficient time; four or five lessons seemed necessary for a satisfactory completion. In order to allow for this, and under the supposition that the other planned activities had not been allotted enough time, two of the original activities were omitted. They were the third, the number of different configurations possible using a fixed number of the shapes cut from card stock, and the seventh, problems involving fixed numbers of straws. These two activities were eliminated because both involved the kind of restriction on constructions which were only sometimes well received by the first grade students in the pilot studies. This added three days to the other activities as needed. The sequencing remained the same.

In preparation for the next day, a new chart which showed the seven kinds of coverings was made from seven pieces of paper, $5\frac{1}{2} \times 8\frac{1}{2}$

inches. The numbering was the same on both charts. The new chart was to serve as a heading under which drawings of coverings could be hung in columns. The new chart was the top line of Figure A.6, page 183.

Day 3: Plan

The teacher reviews with the students the way the chart was made and what it shows, and the rules for making coverings including the promise papers. The she asks each student what kind of covering he made the previous day and what it looked like. She emphasizes occasionally that she does not remember what the student's covering looked like, and finally, when each student has had a turn, asks, "How could we make it so that we could remember what our coverings look like after we were done with them?" Two restrictions are to be placed on the methods suggested: they must not use up the tiles and the memory device should be something that could be hung up. The reason for wanting them to be hangable can be explained by the teacher by showing the students the new chart she has made so that they could hang up under the appropriate category something showing the way their coverings looked.

It is expected that one of the students will suggest drawing pictures of the coverings. This idea satisfies the requirements. If it is not done or suggested by the students, the teacher might suggest tracing the shapes as a way to draw the pictures. It is not necessary that the students trace; freehand drawings are acceptable.

The teacher passes out paper, pens and tiles, reminds the students of the paper promises, and lets them begin. As they work, she should help each student make sure that his drawing corresponds to his covering.

Some students may try the drawing first. This technique can lead to problems since erasers are not available. If a student tries the drawing first and runs into this problem, the teacher should suggest that he "Do it with the tiles first to make sure it works."

As the drawings are finished, the teacher can ask the aide to hang them up. The student should tell the aide the appropriate column.

Day 3: Events

The review and introduction of the problem went as planned. Various ways were suggested to remember what the coverings look like: leave the tiles on the paper, paste the tiles to the paper, tape the tiles together. Mrs. Gornowicz raised the problems with these suggestions and then one student suggested drawing a picture. Mrs. Gornowicz asked how they would be able to draw the right shape for each tile and another student suggested tracing the tiles.

As the students worked, one remarked that two triangles were "the same as" a diamond. She was encouraged to demonstrate this by covering a diamond with two triangles and by covering two triangles with a diamond. Another student tried to trace first and then make a matching covering, but grew frustrated trying to make her covering match the drawing and made a new covering instead. A student who had tessellated the paper with the triangle in the pretest drew that arrangement without using the tiles first.

When a student was done with one drawing and starting on a new covering, the teacher or the author tried to encourage but not force him to try a kind no one else was doing. Not all students finished one drawing. Unfinished papers were saved for the next day.

Day 3: Evaluation

The carry-over from the pretest was evident in the quickness of the students to suggest tracing and in the tessellation with a triangle drawn by the only student in the program who made that response to the pretest.

The activity was slow and difficult for some students. A few were still having the angle problem of the first day and were reluctant to remove the offending square or squares. However, interest and enjoyment seemed good. It was decided to continue the activity until one picture was hung under each category but not longer than two more days.

Day 4: Plan

The teacher reviews the previous day's activities, emphasizing the promise papers and the goal of trying to get at least one picture under each category. She mentions that if a student draws first and makes a mistake he can't erase it, so he should be sure he knows whether an idea will work before he draws it, and that one way to make sure is to try it with tiles first. Then the teacher passes out the unfinished papers and other materials and tells the students to finish what they were working on and to start new coverings when they are done with the old ones. She again emphasizes trying to get at least one picture under each category and suggests that if a student wants to do a kind that has already been done, he should try to use the same shapes in a new way. The students are encouraged to walk over to where the pictures are hanging, if they want to check to see if their's is different from the ones hanging up.

Day 4: Events

The review and introductory remarks went as planned. The students seemed to enjoy the work much more than on the previous day and to find it less difficult.

Coverings by squares only continued to be popular. One student, starting another such covering in a checkerboard arrangement just as were the three already hanging, was asked if she could make her covering different. The teacher hinted that perhaps the arrangement of the blocks in the wall of the room might give her an idea. The student decided that the best way to make her covering different was to insist that her paper be hung with the long side vertical instead of horizontal. She admitted that if hung horizontal it would be the same as the others, but was positive that changing orientation of the paper made her covering different.

At the end of the class, two drawings were hanging under the category squares and triangles, as shown in Figure A.3.

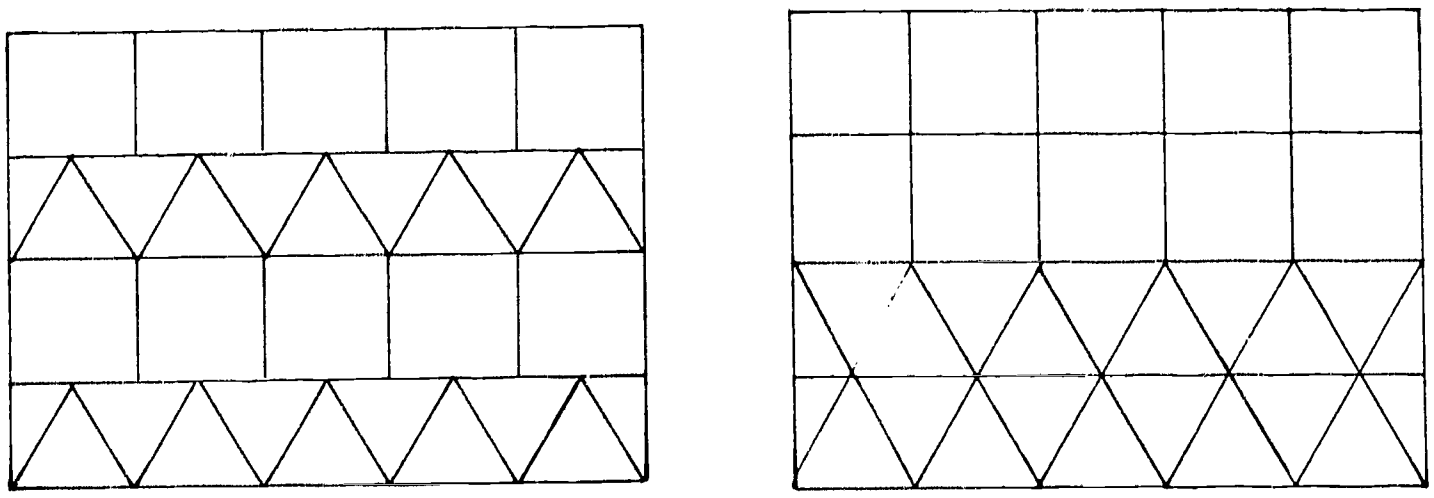


Figure A.3

Two Coverings Having Horizontal Rows

The student who made the one on the left was asked if these two were different. She replied that they were because the one on the left had squares "there" (pointing to the second row) and the one on the right had triangles "there."

The student who made the covering on the right had said that she was making "houses." Her comment was heard by another student who looked at her work and then produced the covering shown in Figure A.4.

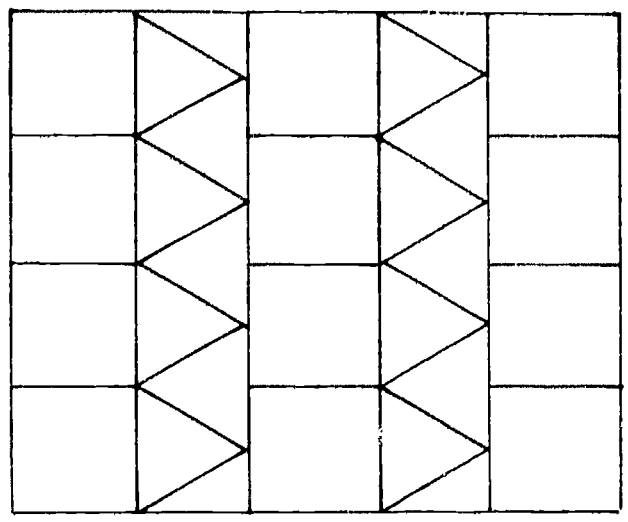


Figure A.4

A Covering Having Vertical Rows

At the end of the period there were pictures hanging under all the categories except two: triangles and diamonds, and all three shapes. When Mrs. Gornowicz asked if anyone was going to do one of these, one of the students replied that the kind with all three shapes was "too hard."

Day 4: Evaluation

Some very interesting coverings were made and drawn during this lesson. The students seemed better able to modify ideas and see

similarities and differences than on previous days.

The recognition of rows in the coverings, done by one student in comparing two drawings, gave the teacher and the author the idea to have the students examine all the coverings for the existence of rows. It happened that all three shapes had been used in rows at least once, and this fact could be pointed out to the students. Then the teacher could ask the students if they thought that using rows could help them do the kinds for which there were still no drawings. This was agreed upon as the plan for the final day of this activity.

Day 5: Plan

The teacher starts the class by having the chart with the drawings on the table where the students can see it. She points to the two drawings compared the previous day and asks for the ways they are similar and the ways they are different. If necessary she introduces the word rows to describe the differences. Then she asks, "What about the other coverings? Which of them are made in rows?" Each covering should be examined.

Then the teacher should say, "What about the two kinds without any pictures under them - do you think you could make a covering with triangles and diamonds or one with all three shapes using the idea of one shape in each row? Who will try it?"

The teacher should also point out that all the coverings using squares only look the same and direct the students' attention to the wall to see if that will give them an idea for how to make a different covering using just squares.

Then she passes out the materials and lets the students start.

Day 5: Events

The introduction went as planned. The students used the phrase "looks like squares" before the word "row" was introduced and seemed to be referring to a similarity to the orderliness and regularity of the checkerboard arrangement of squares when they used the phrase.

The student who made the "houses" covering took up the challenge of using all three shapes. She said, "I'm going to start the same way I did yesterday," and made a row of squares, then a row of triangles, then she added a row of diamonds to the old pattern and finished with the pattern shown in Figure A.5.

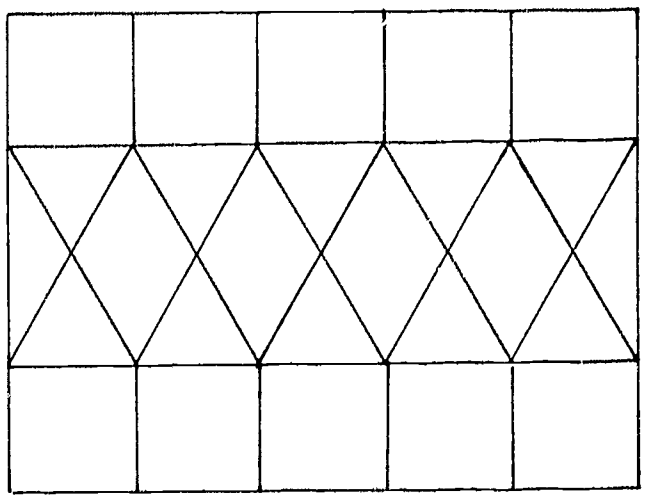


Figure A.5

A Covering Using All Three Shapes

While she was working she said that she didn't see why nobody had made this kind before because it was so simple. The remark was overheard and it was explained to her that something was always simple when you knew how to do it, but that if you know how and someone else doesn't then it isn't simple for them and if you say "It's simple," you may make them feel bad. Not much real understanding of these comments was expected,

but a few minutes later, the same student was heard explaining the principle to the others. The students began calling "simple dimple" a "magic word" which one could not say.

This became somewhat of a game and - anticipating a bit in the journal - it was brought up by the students daily until the end of the program.

One student took up the challenge of the squares. He carefully looked at the wall and worked on his paper, producing after about five minutes the partial covering (a) of Figure A.6. Then he started a second column, as shown in (b) of Figure A.6.

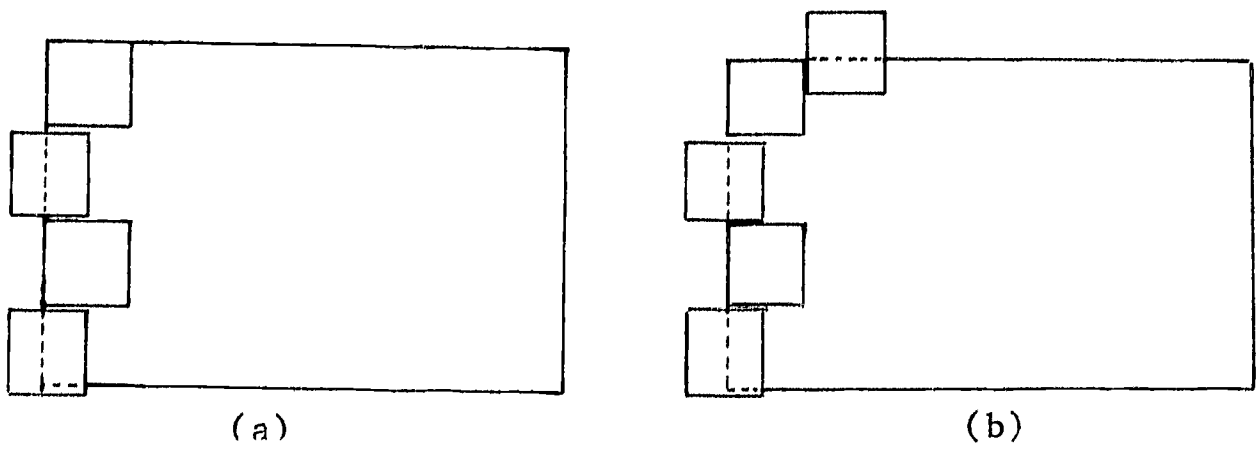


Figure A.6

An Attempted Covering by Squares

With this arrangement he had problems, obviously. He tried for a while, then gave up and did a different kind of covering.

At the end of this period the wall chart looked like Figure A.7, page 183.

Day 5: Evaluation

The row idea worked well and by the end of the day all the categories had pictures under them. It was time to end this activity.

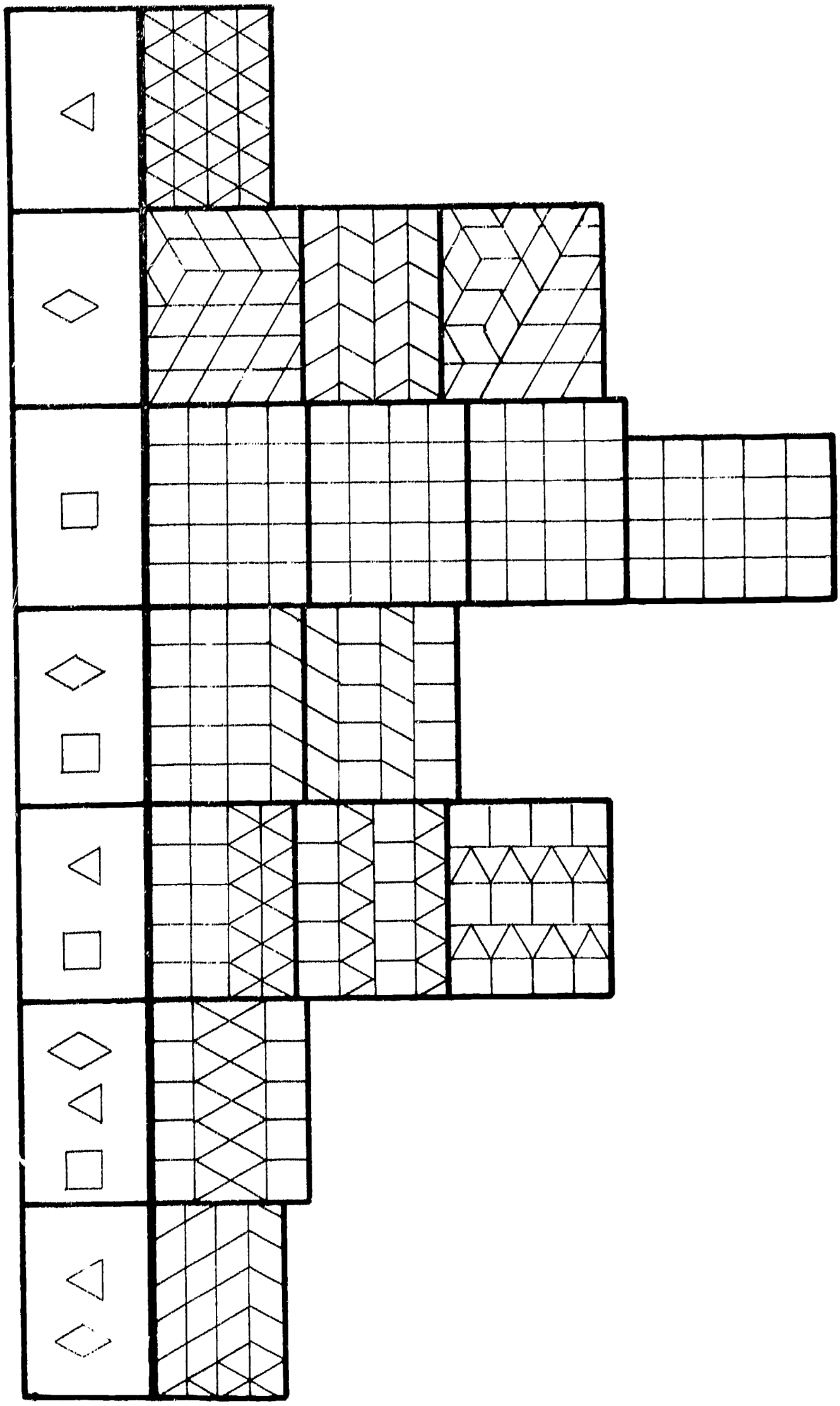


Figure A.1

Summary Chart Made During Activity One

Some students never really did grasp the way around the angle problem; some seemed much more willing by the fifth day to remove tiles and start again.

In talking about the activity for the next day, the teacher and the author both felt that rather than go to the shapes cut from card stock, a material quite similar to the tiles, that it would add more life to the change of activity to go to the straw and pipe cleaner materials. A new sequencing was chosen: straw and pipe cleaner activities (there were two of these in the original plan) making objects from card stock shapes; making same objects from both sets of materials; making new shapes which can tessellate. This sequencing shared with the original the aspect of some later activities recalling earlier ones and seemed equally satisfactory.

ACTIVITY 2: STRAW AND PIPE CLEANER CONSTRUCTIONS

Materials: Plastic drinking straws of various solid colors and three lengths: $10\frac{1}{4}$ inches, $5\frac{3}{4}$ inches, $4\frac{1}{2}$ inches.

White chenille pipe cleaners

Gummed tags

Pens

String

Tape

Six cardboard boxes

The first day or so was to be devoted to making objects from the straws and pipe cleaners. Then when the students became sufficient

familiar with the technique of making the objects, the problem of making objects with Euler lines was to be introduced.

Day 6: Plan

The teacher says, "Today we are going to start something new. We are going to use these (open a box of straws) - do you know what they are? They come in three sizes." The teacher takes one of each size straw and holds all three upright on the table so that the students can see the three different lengths. "They also are different colors," the teacher remarks. The teacher next opens a box of pipe cleaners and says, "We also have these, do you know what they are called?" Then the teacher demonstrates how to use these materials as she says, "We can build things with straws and pipe cleaners. We use a pipe cleaner folded in half and put part of it into the end of one straw and the rest of it into the end of another straw, making sure no pipe cleaner shows. Then we can bend the pipe cleaner to make a corner of any size we like. We can add a third straw to the corner by putting a bent pipe cleaner half in the new straw and half into one of the straws already in the corner." The teacher demonstrates and explains the technique again if necessary stressing the aspect of no pipe cleaner showing when a corner is done. Then she tells the students that each should make the most interesting thing he can think of, using these materials, and says that more than one size straw can be used on one object. She also shows the students an octahedron as an example of one kind of object which can be made. But she makes it clear that the students do not have to copy what she

has done. Then the teacher distributes the straws and pipe cleaners and lets the students begin.

At the end of class, the teacher passes out the cardboard boxes and pens so each student can write his name on a box. Then each student stores his constructions in his box. For those objects too big for the boxes, a gummed tag can be used to label each object with the student's name.

Day 6: Events

The introduction went as planned. As the students worked, several of them had problems with the use of the pipe cleaners; they would put most of the pipe cleaner in one straw and not enough in the second, so the corner would fall apart. Mrs. Gornowicz and the author moved among the students trying to help them and showing them that the corners were more stable if about half the pipe cleaner was put into each straw.

One student asked for scissors to cut the straws. She was told that we had none.

Many of the objects made were animals or swing sets. One student made a square and called it a "box." Then she added legs to it in response to a question about whether her box could stand up.

Two of the students tried to copy the octahedron. One counted the number of triangles in it and proceeded to start making eight separate triangles. At the end of the period, when she had five triangles done, she was overheard saying, "Now how am I going to put these together to get that?" The other student made one triangle, put it on the octahedron, added a straw, again placed his object on the octahedron, added another straw and continued in that fashion.

Day 6: Evaluation

The students' difficulty with the construction technique led the teacher and the author to suspect that introducing the Euler line problem on the second day should be postponed until an appropriate lull, if one occurred, or be given to individual students who seemed ready for it. It also seemed that the attempts to reproduce the octahedron were having such worthwhile results in terms of analysis of the problem and persistence toward an end that it would be contrary to the aims of the program to interfere with these attempts.

Day 7: Plan

Before the students arrive, all the cardboard boxes are placed on the table, one at each student's seat. When the students arrive, the teacher says, "Before you open your boxes, can you tell me what is inside the box? How did you make it? What did you use?" After each student has had a chance to answer, the teacher distributes the materials and lets the students begin work. She tells them that they can finish what they were working on or start something new. She places on the table the octahendron and also a tetrahedron.

The same cleanup procedure should be followed as was used on Day 6.

If there is a general lull, or if one or two students seem to be ready for a new problem, the teacher may introduce the Euler line problem. She should show the students first a simple construction such as a triangle which does have an Euler line and should actually put the string onto the construction as a demonstration for the students. Then she

should try putting string on another construction which does not have an Euler line, such as a square with two diagonals to show the students that not all constructions can have string on them in that way. She can use tape to fasten the string to the constructions. Once the students have the idea and know the rule of one and only one piece of string on each straw, the teacher should let them try to build objects which have an Euler line. She will need to help and encourage the students because this is a very difficult problem.

Day 7: Events

The introduction went as planned. The students became quite engrossed in building, and one or two more took up the problem of reproducing an octahedron, so Mrs. Gornowicz did not introduce the Euler line problem.

Mrs. Gornowicz and the author were quite busy this day helping the students. The student who had made five separate triangles the previous day, made three more this day. She needed much encouragement to take some of them apart in order to fashion an octahedron from them without the octahedron having double straws as edges and being, in the student's eyes, "wrong." Another student made a pyramid on a square base as a start of an octahedron, but then in adding to it, made one vertex with five straws instead of four. She was helped in counting the number of straws at the vertices of her object and on the octahendron, but ended up making an object looking like two pyramids on pentagon bases attached base to base.

Not all students were attempting to copy some model, there were also animals and swing sets still being made by some.

Day 7: Evaluation

It was a good busy day; Mrs. Gornowicz was quite correct in not interferring with the students' work for the purpose of introducing a new problem. The decision was made to use approximately the same plan for the next day.

Day 8: Plan

The students' boxes should be at their seats before they get to the room. The teacher should distribute the materials as soon as the students arrive and let work begin immediately. The same plan for introducing the Euler line problem should be followed as is outlined in Day 7: Plan. The usual cleanup activities should end the lesson.

Day 8: Events

This was another busy, somewhat noisy day. The students were almost as productive as on Day 7, and Mrs. Gornowicz again decided not to introduce the Euler line problem.

By the end of this lesson, all the students had tried to reproduce either the tetrahedron or the octahedron. One student, trying to make a tetrahedron, made a pyramid on a square base. She was handed the octahedron to compare her object with, and after some maneuvering, declared that what she had was "half" of the octahedron. She then began

to say that making the octahedron would be simple, when she remembered the prohibition about the "magic word" and only gave the author a knowing look. This remembering not to say "simple," although not reported in the events of every day, was an everyday occurrence, much to the surprise of Mrs. Gornowicz and the author.

One student made a box or cube; another student made two tetrahedra, a big one and a little one. Other students made other objects such as fans and rakes.

Day 8: Evaluation

Although the activities of the day were worthwhile enough to support Mrs. Gornowicz's decision not to introduce the Euler line problem, the peak of productive mathematical work seemed to have been reached and passed by the end of the lesson. The decision was made to start a new activity the next time.

ACTIVITY 3: SURFACES

Materials: Collection of several hundred each of equilateral triangles, squares, and diamonds (rhombi formed from two equilateral triangles), each $1\frac{1}{2}$ inches per side, cut from white card stock. Each shape was stored in a separate shoe box.

Tape

Pens

Cardboard storage boxes from previous activity

This activity was to be primarily the making of surfaces, both open and closed, both flat and three-dimensional, by taping the cards together edge to edge.

Day 9: Plan

The teacher opens one box of cards, picks up one of them and asks for its name. She does this for each of the three shapes. Then she says, while demonstrating, "We can make things by taping these shapes together like this, edge to edge. Here are some things I made just for fun. (She holds up a tetrahedron, an octahedron, an icosahedron, a cube and a cubo-octahedron.) You can make anything you like; it can be flat or it can stand up. It can be like something I made, or it can be very, very different."

The teacher then distributes the cards and tape to the students and lets them begin work.

At the end of the period, all objects, finished or not, should be stored in the individual storage boxes.

Day 9: Events

As usual, the introduction went as planned. Some students tried to copy the tetrahedron or octahedron, others made cubes and from there went on to make houses. The students became mildly competitive in trying to make things different from those made by the others. This resulted in, among other things, dog houses, churches and barns, all of which would appear identical to the unknowing eye.

One student made a beard: a long strip with parallel sides having width equal to the longer diagonal of the diamond. He made it from

triangles and diamonds.

Another student began to tape the cards together into a flat surface. As she worked she remarked, "I'm doing this just like we did that (pointing to the pictures of the coverings); I'm trying to make it so that no table shows through."

Occasionally a student would become frustrated trying to put the last few cards on a closed surface. As often as possible Mrs. Cornowicz and the author tried to lend an "extra hand" to the students who needed it.

Day 9: Evaluation

The author was pleased that one of the students saw a similarity between this activity and a previous one. Also pleasing was the competitiveness to make different objects, although the way in which the objects were different was in name not shape.

The teacher and the author decided that one more day should be devoted to this activity so that the students could become better acquainted with the technique, then to proceed to the activity of making the same shape with both the cards and the straws and pipe cleaners.

Day 10: Plan

The teacher passes out the individual storage boxes and distributes the materials and lets the students begin work, either starting a new object or finishing one from the previous day. At the end of the period, the objects should be stored in the cardboard boxes.

Day 10: Events

In addition to the boxes, houses and copies of objects presented

by Mrs. Gornowicz, a new structure appeared during this lesson - a crown. Crowns were made by several students, copying the idea of one of them, by taping together a row of squares and joining the first and last square, then taping a triangle to the upper edge of each square. Some of the girls used this idea to make bracelets.

During the lesson the following conversation took place between one of the students and Mrs. Gornowicz:

Student: I like what we're doing today because it's play.

Mrs. Gornowicz: If this is play, what is work?

Student: That (pointing to chart with pictures of coverings) was work.

Mrs. Gornowicz: What's the difference between work and play?

Student: Work is hard and play is fun.

Several other students had heard these remarks, and agreed with the definitions. A discussion followed about how something can be work for one peron and play for another. And, of course, "simple dimple" made its way into this conversation.

Day 10: Evaluation

This was another good day with much activity. It was rewarding for the teacher and the author to see one student begin using an idea, such as the crown construction, and then other students pick up on the idea and modify it to suit their desires. Encouraging this kind of activity was one of the goals of the program.

That one of the students should define work and play was, of course, an unexpected outcome of the program. What surprised the teacher and

the author even more than the presentation of a definition was the concise nature of the definition and the agreement of this definition with the discussions offered by some psychologists and educators.

ACTIVITY 4: SHAPES

Materials: All materials from Activity 2 and Activity 3 except the string.

Through comparison of objects having the same shape but made with different materials, the concept of having the same shape was to be discussed. Then each student was to try to make pairs of objects having the same shape, one of the pair from each kind of materials.

Day 11: Plan

The teacher holds up two tetrahedra, one made of straws and pipe cleaners and one made from the card stock. She asks, "In what ways are these two objects different?" Expected responses to this question are in terms of colors, materials, size, and perhaps open versus closed. The teacher should suggest the first one or two of these properties if the students do not, but it is not necessary for all the properties to be discussed before the teacher asks, "In what ways are these the same?" The desired response to this question is that the objects have the same shape. This response is not likely, but partial descriptions of what it means to have the same shape may be offered; for example, a student might suggest that both objects have triangles. Other examples of partial descriptions could be having the same number of triangles, having points or corners, having the same number of corners. The students might say

that the objects look the same or call them both by the same name. The teacher should not force discussion of the similarities of the two tetrahedra, but should let the discussion go as far as it can naturally. Then she says, "We say these two things have the same shape." Then she should repeat the procedure using two octahedra.

After the second comparison, the teacher says, "Today, let's each make one thing with the cards and then make the same shape thing with the straws and pipe cleaners. You can make any shape you want, but you have to make the exact same shape out of both kinds of materials. If you want to, you can use something already in your box as one of the two things to have the same shape."

The teacher then passes out the materials and lets the students begin. The storage boxes should be left at the side for consultation as needed and for storing the objects at the end of the period.

Day 11: Events

During the search for differences between the two tetrahedra, the students offered the following comparisons: one has many colors and the other is white, one is bigger, one is held together by tape and the other by pipe cleaners, one is made of cardboard and the other of straws and pipe cleaners, you can put your hand through one but not the other. This list includes all the possibilities in the plan and the additional one of the means used to hold the object together. The search for similarities went as expected, with Mrs. Gornowicz introducing the word "shape."

Several students completed a pair by matching objects already made and in the individual storage boxes. Some students did not look in the boxes first, but stated that a such-and-such was in the box and they were making another one out of the other materials.

In addition to the tetrahedra, octahedra and cubes made during the lesson, an animal was made. One student, having a giraffe made from straws and pipe cleaners, taped about six squares in a row to duplicate the long, thin aspect of a straw, and from such rectangles made a flat giraffe. Other than this reinterpretation of the instructions, all the pairs of objects were made with the expected understanding of the notion of shape.

Day 11: Evaluation

Both the teacher and the author were pleasantly surprised by the number of properties which the students compared the two tetrahedra, especially because one of the properties was not anticipated. It was clear that this response was attributable in part to familiarity with the objects and materials. The contrast between the response to this lesson and the response to the comparison activity in the first lesson was so great that it presents a temptation to attribute some of the difference to the participation in the program. This is a somewhat midleading temptation, however, because the objects seem to have, in the view of the teacher and the author at least, more perceptable differences in properties than did the coverings.

It was decided to continue this activity for one more day.

Day 12: Plan

The teacher reviews the activities of the previous day by having the students explain what they had been doing. Then she passes out the materials and lets the work begin. At the end of the hour, all objects should be stored in the storage boxes.

Day 12: Events

The activities pursued were similar to those of the previous day.

One student ran out of ideas for shapes and asked for a suggestion. Since he had previously made a crown from the card stock, it was suggested that he make that shape from straws and pipe cleaners. He thought about that for a while, then said that he knew he could do it if he wanted to, but that it was too hard, so he didn't want to. His running out of ideas seemed paralleled by the actions of some other students.

Day 12: Evaluation

The decision made on the previous day to end this activity with this lesson was confirmed by the actions of the students; they were not nearly as interested or enthusiastic about the activity the second day and some ended work before the timer sounded - a rare occurrence during the program. The final activity, making shapes which tessellate, was to be started the next day.

ACTIVITY 5: SHAPES WHICH TESSELLATE

Materials: All materials from Activity 3 plus white unlined paper $5\frac{1}{2} \times 8\frac{1}{2}$ inches.

For a chart, manilla paper 11 x 18 inches and a black felt-tip pen.

The purpose of this activity was to discover some shapes that could tessellate a planar surface and which could be made from equilateral triangles, squares and diamonds, all having the same side length. A chart showing which shapes could tessellate was to be made.

Day 13: Plan

The teacher has the card stock shapes and tape in front of her, nothing else on the table. She says, "Look at the pictures on the wall. They are of coverings we made a couple of weeks ago. Do you remember what shapes we used in the coverings?" The students will either remember or look at the pictures and respond with "triangles," "squares," and "diamonds." Then the teacher says, "Some of the coverings used more than one shape, but some used only one shape. What is one shape we can use to cover a whole paper?" One of the three basic shapes will probably be offered as an answer. The teacher then repeats the question until it has been brought out that each of the three basic shapes can cover by itself. Then the teacher demonstrates and says, "These boxes have triangles, squares, and diamonds in them. We know that triangles alone can cover and that squares alone can cover and that diamonds alone can cover. Suppose we tape a triangle edge to edge with a square; we get a new shape. We can get another new shape by taping a square to a diamond or by taping two triangles to opposite sides of a square. We can make lots of new shapes this way. Now some of these shapes might be able to cover a piece of paper. We could find out by making many of the same

shapes and then trying to fit them together to cover the paper. Do you remember the rules for covering?"

The students may need to be reminded of the rules of no overlapping of shapes, no paper showing, and going over the edge of the paper being permitted. When this review is completed, the teacher should repeat, "Each of you choose one shape and make many of that one shape, and then see if you can cover the piece of paper with the shapes." She then passes out the materials and lets the students begin.

This activity will be somewhat more difficult than the previous ones, so the teacher should be prepared to re-explain the task and to help and encourage the students more than usual. The new shapes can be stored in the individual storage boxes at the end of the lesson.

Day 13: Events

The introduction went as planned, but evidently not enough explanation of the task was planned, because each student needed a re-explanation. This need caused the first few minutes of the activity to be rather hectic, but then everything settled down and the students began to work.

Five new shapes were tried, one by two students. Four of the shapes were made to tessellate; the fifth was impossible. The student who made the impossible shape became frustrated and began taping triangles, squares and diamonds to her shape to make one large covering.

One student, having covered the piece of paper with a shape, seemed uninterested in the suggestion of finding a new shape which also covered, so it was suggested that she could draw a picture of her covering. This

she liked, and she began tracing one of her shapes to make the picture. Mrs. Gornowicz noticed that she would add to her picture the lines which were boundaries of the old shapes from which her new shape was made. After a bit of explanation, the student decided that those extra lines should be left out.

The other students also liked the idea of drawing a picture of their coverings. Time ran out before everyone could finish, so the shapes and papers were saved for the next day.

Day 13: Evaluation

The teacher and the author had miscalculated the degree of explanation needed for this activity, but otherwise, the activity went well.

The making of pictures had not been planned but seemed like a natural activity to suggest and was quickly taken up. Although there was no plan to hang the individual pictures on the wall, a chart was planned to display an example of each shape which could tessellate. It was decided to make the chart part of the review during the next lesson.

Day 14: Plan

The teacher reviews with the students the activities of the previous lesson: the making of shapes which can cover. Then the teacher should suggest that they make a chart on which they could tape one of each kind of shape that could cover. As she makes the chart, the teacher should ask each student if he would contribute one of his shapes to put

on the chart. She should also ask the students to help choose a name for the chart. The chart, when finished, should look something like Figure A.8 (using the shapes made the previous lesson).

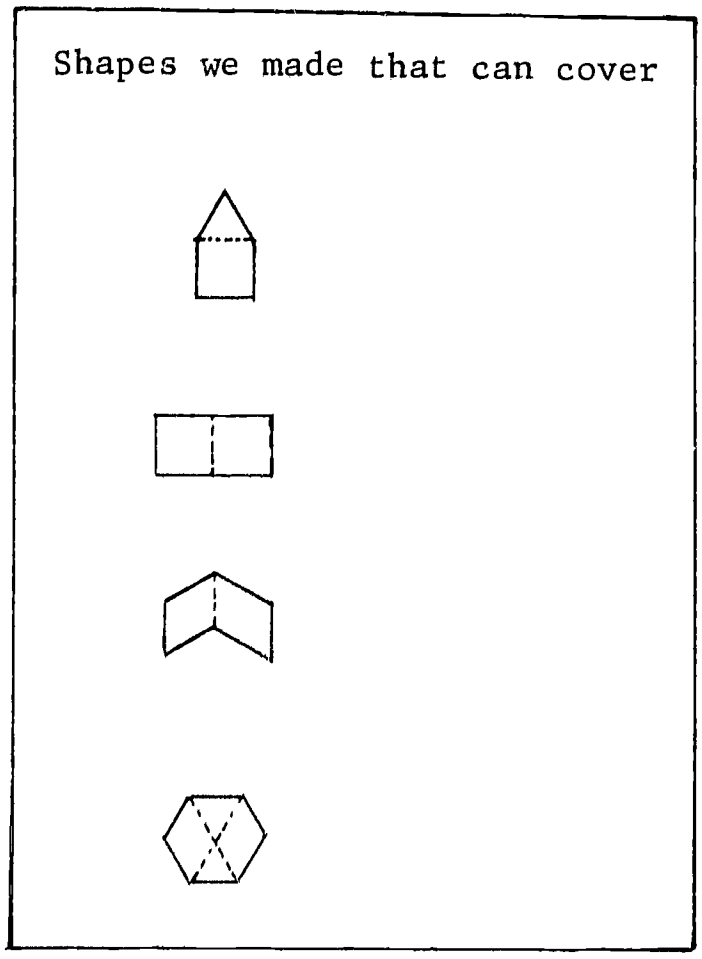


Figure A.8

Proposed Summary Chart for Activity Five

When the chart is done, the teacher should say, "Today let's see if we can find more new shapes to put on our chart." Then she should pass out the materials and let the students begin work. At the end of the period, shapes and papers can be stored in boxes.

Day 14: Events

The chart was made and new shapes were added to it when the student

showed Mrs. Gornowicz or the author how the new shape could be made to cover a piece of paper.

The student who did not make a tessellation during the previous lesson made hexagons from three diamonds during this lesson. She insisted that one of her hexagons be put on the chart even though a hexagon was already on the chart, because the one already up was made of two diamonds and two triangles, so hers was different.

Some of the students who did interesting work the previous day seemed less enthusiastic about the task this day. When the lesson was finished, the chart looked like Figure A.9.

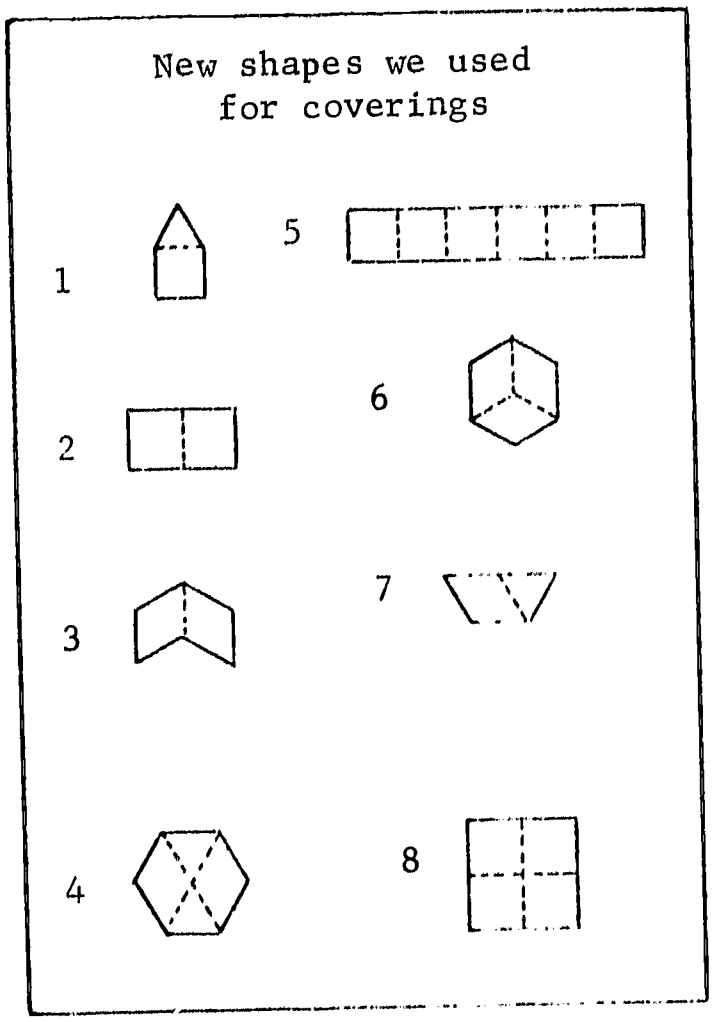


Figure A.9

Summary Chart Made During Activity Five

Day 14: Evaluation

Because the enthusiasm was falling off and because there was only one lesson left in the program, the teacher and the author decided it would be best to have the next lesson be a free period with all the materials available.

Day 15: Plan

The teacher distributes all the materials from all the activities except Activity 1 and tells the students that they can make anything they want. At the end of this period, objects are to be put in the storage boxes as usual, no mention is to be made of the fact that this is the last lesson.

Day 15: Events

The students spent most of the time constructing objects from either the card stock shapes or the straws and pipe cleaners. One student was posed the problem of making, with straws and pipe cleaners, an object having two triangles and three squares. He strugged with the problem for quite a while and then gave up and started a new project. By chance, another student had constructed a triangular prism, which was borrowed and shown to the first student. He looked it over and decided that it satisfied the requirements of the problem, and then returned to his new project.

Some of the students made objects of the same shape with both kinds of materials or of two sizes of straws.

Day 15: Evaluation

The day without definite goals was a good break for the students and seemed to perk up their interest a bit. Had the program continued beyond fifteen lessons, another day or two of relatively free activity should have preceded the introduction of new problems.

APPENDIX B

FACE VALIDATION MATERIALS AND RESULTS

FACE VALIDATION MATERIALS AND RESULTS

This appendix contains the list of proposed criteria with examples of each which were judged by the seven Professors of Mathematics from the University of Wisconsin. Also in the appendix are the glossary defining some of the terms in the criteria, a sample scoring sheet used by the author to record each professor's judgments, and the results of the face validation proceedings.

The seven professors were Dr. Richard A. Brualdi, Dr. Joshua Chover, Dr. Donald Crowe, Dr. Simon Hellerstein, Dr. Mary Ellen Rudin, Dr. Hans Schneider, and Dr. Melvin C. Thornton.

THE PROPOSED CRITERIA

Part A

After the task has been outlined by the teacher, during pursuit of the activity the student:

1. In the absence of a specific stated mathematical goal, verbally introduces some appropriate goal; and or exhibits goal-directed behavior with respect to some appropriate goal.

   e.g.: The teacher has not suggested the goal of maximizing the number or triangles traced, and the student states that as his goal; or, the student, under the same circumstances, makes several arrangements of tracings and counts the number of copies in each tracing.

2. States an appropriate unstated property of the activity or its product.

   e.g.: The student says, "You can make a pattern with the tracings."

3. Demands motivation not intrinsic to the task as stated.

   e.g.: The student asks what reward he gets for achieving the goal, or asks why he has to do the task.

4. Seeks approval of the teacher for his actions.

   e.g.: The student looks questioningly at the teacher after making each tracing and does not begin the next tracing

until the teacher has said some words of praise or recognition.

5. Conjectures, states, or demonstrates a possible relationship between some appropriate property of the activity and/or products of the current task and some appropriate property of the activity and/or products of either that same task or some previous task; and/or investigates a relationship of the above type.

   e.g.: The student says, "Maybe the number of tracings I can fit on the paper has to do with the number of sides I draw," or, "Look, this pattern of all triangles is also a pattern of that shape I made from two triangles." The student, having seen a tesselation made with triangles, makes a shape with four triangles and then tries to tesselate with that shape.

6. Exhibits goal-directed behavior with respect to a goal stated or induced by the teacher.

   e.g.: The teacher suggests the goal of maximizing the number of triangles; the student periodically stops tracing the cardboard triangle to count the number of tracings on the paper.

7. Without having pursued the activity outlined by the teacher, verbally suggests an appropriate modification and/or extension of the task; and/or exhibits goal-directed behavior with respect to an appropriate modification and/or extension of the task; and/or conjectures, states, demonstrates, or

investigates a possible relationship between some appropriate property of the activity and/or products of an appropriate modification and/or extension of the task and some appropriate property of the activity and/or products of either that same task or some previous task.

e.g.: As soon as the teacher has outlined the task, the student says, "I'd rather do a square than a triangle." Under the same circumstances, the student draws a large circle and tries to fill it in with triangles.

8. Conjectures, states, or demonstrates a possible generalization; and/or attempts to generalize.

e.g.: The student says, "I bet that putting the tracings in a pattern always helps you get more." Or, the student, having dissected the square and the rhombus into triangles, tries to dissect the circle into triangles.

9. Achieves, states or demonstrates an appropriate mathematically elegent product or result.

e.g.: The student tesselates the paper with the triangle.

Note: Any modification or extension of the task by the teacher is considered as an outline of a new task by the teacher.

Part B

After the student has pursued the task as outlined by the teacher, the student:

10. Verbally suggests an appropriate modification of the task; and/or exhibits goal-directed behavior with respect to an appropriate modification

of the task; and/or conjectures, states, demonstrates, or investigates a possible relationship between some appropriate property of the activity and/or products of an appropriate modification of the task and some appropriate property of the activity and/or products of either that same task or some previous task.

e.g.: Having tried to maximize with a triangular shape, the student asks if there are any new shapes he can try, takes the new shape, and tries to maximize the number of copies of it he can get on thepaper.

11. Verbally suggests an appropriate extension of the task; and/or exhibits goal-directed behavior with respect to an appropriate extension of the task; and/or conjectures, states, demonstrates, or investigates a possible relationship between some appropriate property of the activity and/or products of an appropriate extension of the task and some approrpiate property of the activity and/or products of either that same task or some previous task.

e.g.: The student, having tried to maximize with a triangle, says, "I'm going to see what kind of shapes I can make with these triangles." Or, under the same circumstances, the student counts out ten triangles, makes a surface using all of them, counts the number of boundary edges of the surface, and repeats the latter two actions.

12. Does not modify or extend the task in any way.

    e.g.: The student completes the task once and stops work.

13. Requests a modification or extension of the task, but does not offer one.

    e.g.: The student says, "Can I do something different with the triangles now, or do I have to do this again?"

If, while working on a modification or extension of the task made by himself, the student does any of the first nine actions, would these actions be at least as representative of creative activity under these new circumstances as they would be if they occurred after the teacher had outlined the task?

GLOSSARY (LOGICAL ORDER)

ACTIVITY: a set of actions performed, possibly incorporating some materials.

e.g.: The actions of placing the triangle on the paper, tracing the triangle, moving the triangle to a new position, cutting out a triangle, putting all triangles in a pile, and counting the triangles in the pile are some of the actions which could make up the activity of "tracing triangles on the paper and cutting them out."

PRODUCT: any result of an activity.

e.g.: A pile of ten triangles and some scrapes of paper, or a piece of paper tesselated by a triangle.

GOAL: a set of values for some variables of an activity or a product.

e.g.: The number of triangles should be maximum, or the arrangement of triangles should be a pattern.

(Note: The definition does not include any statement of degree of motivation on the part of any person toward achievement of the goal.)

OUTCOME: any activity or product for which a goal is set.

e.g.: Referring to the definition of "goal," the outcomes are number of triangles and arrangement of triangles.

TASK: activity plus products plus any specified goals.

e.g.: Tracing one cardboard triangle as many times as one can on a piece of paper, then cutting them out.

APPROPRIATE: related to the task and mathematical.

STATES: describing any activity, product, or goal spoken of by the teacher in the outline of the task or otherwise introduced into the task by the teacher.

MODIFICATION OF THE TASK: a new task similar to the previous in at least one activity and product.

e.g.: Fitting as many cardboard triangles onto a piece of paper as one can, next tracing each one, then cutting them out.

EXTENSION OF THE TASK: a new task, differing from the previous in both activity and product, which has a product of the previous task incorporated into the new activity.

e.g.: Taping triangles together to form surfaces.

GOAL-DIRECTED BEHAVIOR: systematic and/or sequential repetitions of some activity, or activities, interspersed with several comparisons of the outcome with the goal.

e.g.: If the goal is maximum number of specific number of triangles on the paper, tracing of the cardboard triangle periodically halted while one counts the number of triangles.

ATTEMPTS TO GENERALIZE: sets or ascertains that the values of some appropriate variables in situation B are equal to the values of those variables in situation A, and then ascertains whether another appropriate variable in situation B has value equal to its value in situation A.

e.g.: Having seen that a patterned arrangement of triangles yields the maximum number of triangles per sheet of paper, one attempts to arrange rhombi in a pattern in order to achieve the same goal, i.e., maximizing the number of rhombi from the sheet of paper; or, having seen that the diamond is made up of two equilateral triangles, one attempts to dissect the square into equilateral triangles.

INVESTIGATES A POSSIBLE RELATIONSHIP BETWEEN A AND B (A and B are appropriate properties, not necessarily distinct): systematically and/or sequentially:

1. changes the value of a property of A and ascertains the resulting value of a property of B, or
2. perform some activity involving both A and B and ascertains the resulting value of a property of some product of that activity.

e.g.: 1. A is the cardboard triangle, the variable is its orientation with respect to the paper; B is also the cardboard triangle, the variable is whether it fits onto a blank part of the paper.

2. One tries to fit the cardboard triangle (A) into the diamond pattern on paper (B), draws the shorter

diagonal of the diamond, and fits the triangle into both triangular halves.

SCORING SHEET FOR FACE VALIDATION

Proposed Criterion	Yes	Maybe	No
1. introduces			
goal-directed behavior			
2.			
3.			
4.			
5. conjectures			
states			
demonstrated			
investigates			
6.			
7. suggests modification			
suggests extension			
goal-directed behavior: modification			
goal-directed behavior: extension			
conjectures modification			
conjectures extension			
states modification			
states extension			
demonstrates modification			
demonstrates extension			
investigates modification			
investigates extension			
8. conjectures			
states			

SCORING SHEET FOR FACE VALIDATION (CON'T.)

Proposed Criterion	Yes	Maybe	No
8. demonstrates			
attempts a generaliztion			
9. achieves			
states			
demonstrates			
10. suggests			
goal-directed behavior			
conjectures			
states			
demonstrates			
investigates			
11. suggests			
goal-directed behavior			
conjectures			
states			
demonstrates			
investigates			
12.			
13.			

It had been decided that a weighted sum of 5 or more was necessary for the acceptance of a criterion. By this standard, proposed criteria 1, 2, 5, 8, 9, 10, and 11 were accepted and proposed criteria 3, 4, 6, 7, 12, and 13 were rejected. Each Professor answered "yes" to the question following proposed criterion 13. A summary of these results is given in Table B.1.

another situation in which the criterion has been satisfied is not contained in any single one-minute segment, but occurs over several minutes, you will score a "1" for that minute during which the sum of the evidence becomes sufficient.

The tapes you will score will show one of two tasks. One involves a triangle and one involves tiles in the shape of a trapezoid or half-hexagon. The names "triangle" and "trapezoid" can serve to distinguish between the tasks.

The materials for the "triangle" tasks are (1) white 5 1/2" x 8 1/2" paper, (2) a cardboard triangle 2" per side, (3) a black marker, (4) a scissors, (5) a cardboard square and a cardboard diamond 2" per side each.

The task is posed to the student as follows: The adult ascertains that the student knows the name "triangle." Then she asks, "How many times do you think you could trace this triangle onto this piece of paper so that you could cut the triangles out?"

The materials for the "trapezoid" task are (1) plastic trapezoid tiles in the shape (a) of the figure and (2) three each of white cardboard forms with black indentations in one of seven shapes:

(a) circle, diameter 2 1/2"

(b) square, side 2 1/16"

(c) trapezoid, shape (b) of the figure

(d) trapezoid, shape (c) of the figure

each criterion can be met or fail to be met regardless of whether another criterion is met or not. If the evidence that a certain criterion has been satisfied is not contained in any single one-minute segment, but occurs c er several minutes, you will score a "1" for that minute during which the sum of the evidence becomes sufficient.

The tapes you will score will show one of two tasks. One involves a triangle and one involves tiles in the shape of a trapezoid or half-hexagon. The names "triangle" and "trapezoid" can serve to distinguish between the tasks.

The materials for the "triangle" tasks are (1) white 5 1/2" x 8 1/2" paper, (2) a cardboard triangle 2" per side, (3) a black marker, (4) a scissors, (5) a cardboard square and a cardboard diamond 2" per side each.

The task is posed to the student as follows: The adult ascertains that the student knows the name "triangle." Then she asks, "How many times do you think you could trace this triangle onto this piece of paper so that you could cut the triangles out?"

The materials for the "trapezoid" task are (1) plastic trapezoid tiles in the shape (a) of the figure and (2) three each of white cardboard forms with black indentations in one of seven shapes:

(a) circle, diameter 2 1/2"

(b) square, side 2 1/16"

(c) trapezoid, shape (b) of the figure

(d) trapezoid, shape (c) of the figure

TABLE B.1

RESULTS OF FACE VALIDATION

Proposed Criterion	Yes	Maybe	No	Weighted Sum
1. introduces	7			7
goal-directed behavior	7			7
2.	6	1		6.5
3.		1	6	.5
4.			7	0
5. conjectures	6	1		6.5
states	6	1		6.5
demonstrates	7			7
investigates	7			7
6.	1	3	3	2.5
7. suggests modification	3	3	1	4.5
suggests extension	3	3	1	4.5
goal-directed behavior: modification	3	3	1	4.5
goal-directed behavior: extension	3	3	1	4.5
conjectures modification	3	3	1	4.5
conjectures extension	3	3	1	4.5
states modification	3	3	1	4.5
states extension	3	3	1	4.5
demonstrates modification	3	3	1	4.5
demonstrates extension	3	3	1	4.5
investigates modification	3	3	1	4.5
investigates extension	3	3	1	4.5
8. conjectures	6	1		6.5
states	6	1		6.5

TABLE B.1 (CON'T.)

	Proposed Criterion	Yes	Maybe	No	Weighted Sum
8.	demonstrates	7			7
	attempts a generalization	7			7
9.	achieves	6	1		6.5
	states	6	1		6.5
	demonstrates	6	1		6.5
10.	suggests	7			7
	goal-directed behavior	7			7
	conjectures	6	1		6,5
	states	6	1		6.5
	demonstrates	7			7
	investigates	7			7
11.	suggests	7			7
	goal-directed behavior	7			7
	conjectures	6	1		6.5
	states	6	1		6.5
	demonstrates	7			7
	investigates	7			7
12.			2	5	1
13.			3	4	1.5

APPENDIX C

MATHEMATICAL PROBLEMS USED FOR PRETEST AND POSTTEST

MATHEMATICAL PROBLEMS USED FOR PRETEST AND POSTTEST

In each testing situation, the student worked alone on the problem in the presence of the author and his actions were videotaped. As the student worked, all remarks he made of a mathematical nature were repeated by the author so that the remark could be clearly heard by the scorers later. This was necessary because the students often whispered. If the student made some mathematical claim or posed some mathematical question the author responded, "That sounds interesting, but how can you know for sure? Can you show me?"

The Pretest Problem

The materials for the pretest problem were one pair of students' scissors; one black felt-tipped marker; one equilateral triangle, one square, and one rhombus of cardboard, each 2 inches per side; and white paper 5 1/2 x 8 1/2 inches. The table at which the student worked was arranged as in Figure C.1.

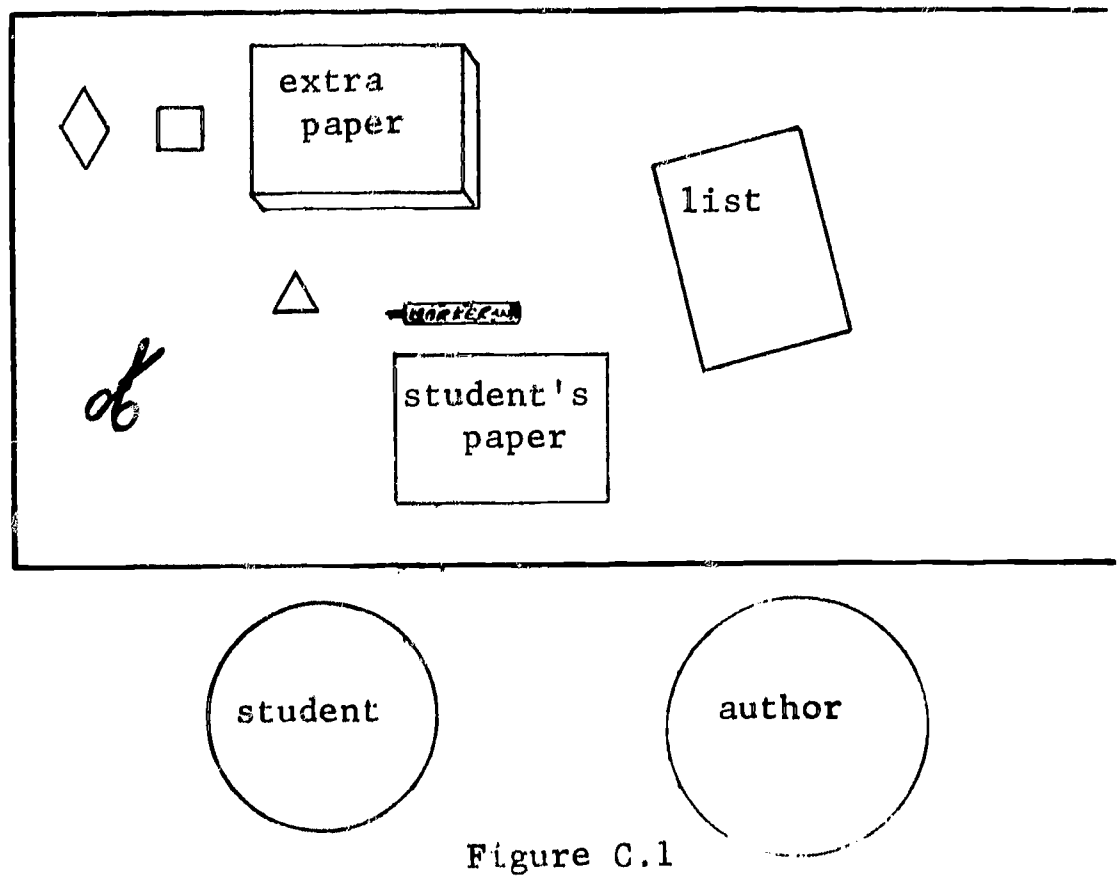


Figure C.1

Arrangement of the Table at the Pretest

The procedure was as follows: First, the author made sure the student knew the name triangle for the shape in front of him. Then the author asked, "How many times do you think you could trace this triangle (point) on this piece of paper (point) so that you could cut the triangles out?"

The author paused, allowing time for the student to guess. If there was a guess, on the paper marked "list" in the diagram, the author wrote the student's name, the word "guess," and then the number guessed followed by a question mark, which the author told the student, "means it's only a guess." If there was no guess, the author said, "Would you like to try and see how many you can get?"

The student would then begin. If the student did not trace the triangle, the author would say, "Why don't you use this triangle as a pattern, so that all the triangles you make will be the same size and the same shape?"

Whenever the student stopped his tracing activity as if he were done with the task, he was asked, "Is there room for any more triangles on the paper?" If he answered "yes" then he was encouraged to add them. If he answered "no," he was asked how many triangles he had traced. The author then wrote "drew" on the list, and recorded the number drawn.

The student was then given a choice between trying to get "even more" triangles on another piece of paper or cutting out those he had drawn. If he chose to draw again the same procedure was followed, including the same choice at the end of the drawing, depending upon the

the time available. The second time the student was also given the choice of returning to his room. (Some students never cut out the triangles.)

If the student chose to cut, he was allowed to proceed, and was asked whenever he stopped as if he were done cutting, "Are there any more triangles left in the paper?" If he answered "yes" he was encouraged to cut them out. If he answered "no," he was asked how many triangles he had cut. The author then wrote "cut" on the list, and recorded the number cut. If the number cut was larger than the number drawn, the student was asked "Do you know where the extra triangles came from?" Depending on the exact circumstances and the time available, the student was asked if he wished to draw again, cut triangles on a paper already drawn, or return to his room.

The Posttest Problem

The materials for the posttest problem were plastic trapezoid tiles in the shape (a) of Figure C.2 and three each of white cardboard forms with black indentations in one of seven shapes:

(a) circle, diameter 2 1/16"

(b) square, side 2 1/16"

(c) trapezoid, shape (b) of Figure C.2

(d) trapezoid, shape (c) of Figure C.2

(e) regular hexagon, side 1 3/8"

(f) equilateral triangle, side 2 1/16"

(g) diamond, side 2 1/16".

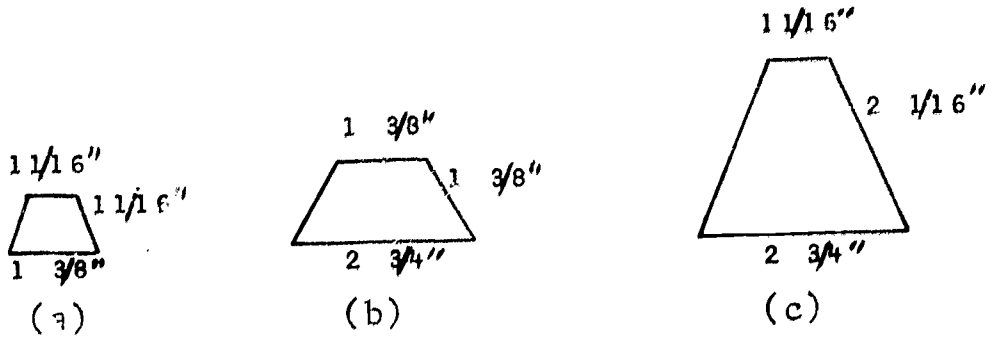


Figure C.2

Three Trapezoids Used in the Posttest Problem

The table at which the student worked was arranged as in Figure C.3.

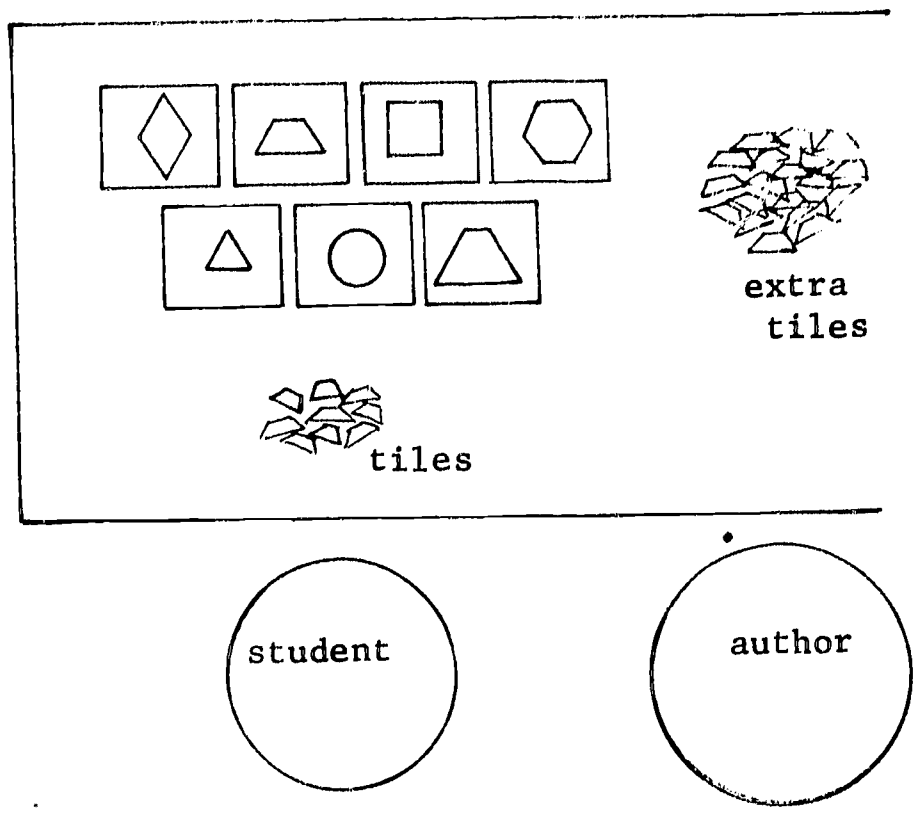


Figure C.3

Arrangement of the Table at the Posttest

The author explained the task to the student as follows: "Some of these block shapes can be covered using these tiles and some can't. Can you find out which ones work by filling up the black spaces and show me why the othere ones won't work? Would you like to guess which ones can be filled up? It won't count against you if you guess wrong."

The student then either made guesses or did not. Then the author said, "Okay, let's see which ones work. You can do them in any order--which one do you want to do first?"

The student picked a shape and began. If he worked for several minutes but could not cover the shape, the experimenter said, "Are you having a problem with that one? Do you think it won't work, no matter how you put the tiles in? Is there a reason it won't work, or do you just feel that it won't?" After each question the student was given a chance to answer. Then the student was encouraged to try another shape. Each student tried all seven shapes. When he was all done he was asked whether he would like to try again on any of the ones that did not work. He then either tried some shapes again or declared himself satisfied and returned to his room.

Occasionally the task had to be more fully explained to the student: "When you fill up the block shape you can't cover any of the white, and none of the black can show."

APPENDIX D

SCORING THE VIDEOTAPES

SCORING THE VIDEOTAPES

This appendix presents the written materials given to the scorers prior to the first training session explanation of their task.

SCORING THE VIDEOTAPES

Your job will be to score the behavior recorded on videotapes according to whether or not it satisfies each of six criteria. (A list of these criteria, explanatory notes, and a glossary are appended to this paper.) The scenes on the tapes are of first-grade students working individually on a mathematics problem in the presence of a adult. The videotape will be run for exactly one minute at a time, after which there will be a pause long enough for you to fill out a scoring sheet. (A sample scoring sheet is appended to this paper.) The minutes will be scored in the order in which the behavior occurred. Because the utmost concentration will be needed to catch all the actions on the tapes, it is strongly suggested that no scorer try to record scores while the tapes are running, but wait until the pauses to do the marking necessary.

The actions of the student during any particular minute may or may not satisfy one of the six criteria. If none of the actions during the minute satisfies a particular criterion, you will put a "0" in the box corresponding to that criterion and that minute. If one or more actions during the minute satisfy a particular criterion, you will put a "1" in the box corresponding to that criterion and that minute. The actions of the student during the minute are to be evaluated on each of the six criteria independently;

each criterion can be met or fail to be met regardless of whether another criterion is met or not. If the evidence that a certain criterion has been satisfied is not contained in any single one-minute segment, but occurs over several minutes, you will score a "1" for that minute during which the sum of the evidence becomes sufficient.

The tapes you will score will show one of two tasks. One involves a triangle and one involves tiles in the shape of a trapezoid or half-hexagon. The names "triangle" and "trapezoid" can serve to distinguish between the tasks.

The materials for the "triangle" tasks are (1) white 5 1/2" x 8 1/2" paper, (2) a cardboard triangle 2" per side, (3) a black marker, (4) a scissors, (5) a cardboard square and a cardboard diamond 2" per side each.

The task is posed to the student as follows: The adult ascertains that the student knows the name "triangle." Then she asks, "How many times do you think you could trace this triangle onto this piece of paper so that you could cut the triangles out?"

The materials for the "trapezoid" task are (1) plastic trapezoid tiles in the shape (a) of the figure and (2) three each of white cardboard forms with black indentations in one of seven shapes:

(a) circle, diameter 2 1/2"

(b) square, side 2 1/16"

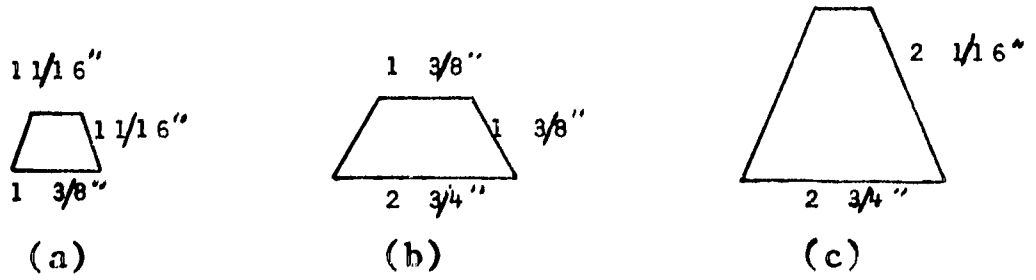
(c) trapezoid, shape (b) of the figure

(d) trapezoid, shape (c) of the figure

(e) regular hexagon, side 1 3/8"

(f) equilateral triangle, side 2 1/16"

(g) diamond, side 2 1/16".



The task is posed to the student as follows: The adult says, "Some of these shapes can be filled in with these tiles and some can't. Can you find out which ones can by filling them up and show me why some of the shapes can't be filled up using these tiles?"

THE CRITERIA

1. Introducing a goal: In the absence of a specific stated mathematical goal, the student either verbally suggests or exhibits goal-directed behavior with respect to some appropriate goal.
2. Identifying a property: The student states an appropriate unstated property of the task.
3. Seeking a relationship: The student conjectures, states, demonstrates, or investigates a possible relationship between (A) some appropriate property of the task he is pursuing and (B) either some other appropriate property of the same task or some appropriate property of some different task.
4. Seeking a generalization: The student conjectures, states, demonstrates, or attempts a possible appropriate generalization.
5. Reaching a mathematically elegant product: The student achieves, states, or demonstrates a mathematically elegant and appropriate product.
6. Modifying the task: After having pursued the task as outlined by the teacher, the student either verbally suggests or pursues an appropriate unstated modification of the task.

GLOSSARY

(logically ordered)

ACTIVITY: a set of actions performed, possibly incorporating some materials.

PRODUCT: any result of an activity.

PROPERTY: a dimension of an object, action or event.

VALUE: a specific instance of a property.

GOAL: a set of values for some properties of an activity or product.

OUTCOME: any activity or product for which a goal is set.

GOAL-DIRECTED BEHAVIOR: systematic or sequential repetitions of some activity, or activities, interspersed with several comparisons of the outcome with the goal.

TASK: activity plus product plus any specified goals.

APPROPRIATE: related to the task and mathematical.

STATED: describing any activity, product, goal, or property spoken of by the teacher or otherwise directly or indirectly introduced into the task by the teacher.

RELATIONSHIP: the mode in which two or more objects, actions, or events, or the properties thereof, stand to one another.

INVESTIGATES A POSSIBLE RELATIONSHIP BETWEEN (A) AND (B): systematically or sequentially either (1) changes the value of an appropriate property of (A) and ascertains the resulting value of an appropriate property of (B), or (2) perform some activity

involving both (A) and (B) and ascertains the resulting value of an appropriate property of some product of that activity.

GENERALIZATION: an inference pertaining to each and all of a class of objects, actions, or events, often based on the truth of the inference when applied to specific members of the class.

ATTEMPTS A GENERALIZATION: sets or ascertains that the values of some appropriate properties of one object, action, or event, (A), are equal to the values of those properties exhibited by a second object, action, or event, (B), (i.e. (A) and (B) are members of the same class); and then ascertains whether on another appropriate property (A) and (B) have equal values.

MOFIFICATION OF THE TASK: a new task arising from the previous task either (1) through a change in at least one but not all of the parts of the task: activity, product, and goal, or (2) through the incorporation of a product of the previous task into the activity of the new task.

EXPLANATORY NOTES

1. Examples of PROPERTY: shape; color. Corresponding examples of VALUE: round, triangle, square; red, green.
2. The definition of GOAL does not include any statement of degree of motivation on the part of any person toward achievement of the goal.
3. In the definition of INVESTIGATES A POSSIBLE RELATIONSHIP BETWEEN (A) AND (B), the (A) and (B) are not necessarily distinct.
4. The definition INVESTIGATES A POSSIBLE RELATIONSHIP BETWEEN (A) AND (B) includes two specific kinds of RELATIONSHIP: (1) causal, changing this causes a change in that; and (2) hierarchical, this is a part of that or this and that are both part of some third thing.
5. In the definition of GENERALIZATION, the inference can be a RELATIONSHIP.

SCORING SHEET

STUDENT_____ SCORER_____

	GOAL	PROPERTY	RELATION-SHIP	GENERALI-ZATION	PRODUCT	MODIFI-CATION
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						
11.						
12.						
13.						
14.						
15.						
16.						
17.						
18.						
19.						
20.						